**Tutorial Class 04 – Discrete Conditional Probability Models**

1.

We know that

Since ( and are independent),

Finally, since , .

Notice that we brought in terms of an actual value in the numerator, but not in the denominator. Think about why this is. The value for will change depending on the value of in the numerator. However, in the denominator, we want to find the total probability that . This is not something that changed based on the value of . As such, it would have led to incorrect results had we changed the denominator.

The distribution of is still binomial. Think of this like have two separate experiments, tossing a coin times and trying to find the number of heads and tossing a coin times and trying to find the number of heads. The combined tossing of times and trying to find the number of heads is still binomial.

There are a few ways to make this equation easier.

* . This term is common in the numerator and denominator so it gets cancelled out.
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We have previously seen experiments where we picked a total of balls from red balls and blue balls and tried to find the probability of picking red balls. We labelled those experiments ‘Hypergeometric Distributions’ and simply used a formula. This is where that formula came from.

2.

Since and count the number of s and s we can get respectively,

and

We need to find .

If we were looking at either or separately, both distributions would just be binomial distributions, i.e. and

However, if consider the value of first, the value of will be limited since . As such,

For , two things have changed. Firstly, the number of possible trials has become , since the maximum value of is reduced once we know the value of . Additionally, we know that a occurred was rolled exactly times, i.e. the other rolls did not land a . Thus, the probability of getting a is increased to .

3. a)

b)

For ,

For ,

For ,

is a derived random variable. Thus,

c)

Let be the event the experiment ends with a success, i.e. there is at least success in attempts. Since is the time required to successfully receive the packets, we assume that must occur. Since the maximum number of attempts is limited to , this must be a conditional truncated geometric distribution.

The probability that there are successes in attempts is .

4. a)

This question deals with the law of total expectation.

Let and .

Say the prisoner picks the first door. In this case, whatever the expected value of is, the value will increase by . The actual expected value will remain the same.

Similarly,

However, if the prisoner picks the third door, he immediately reaches freedom.

b)

Now we deal with a situation in which the expected value of will change depending on which door is picked. Initially,

Again, if the prisoner chooses the third door he immediately reaches freedom so,

If the prisoner chooses the first door, the expected value increases by . However, once back in the cell, the prisoner will only choose between the other two doors. Thus,

Similarly,

5. a)

We know that we can find the joint PMF using conditional PMFs.

Notice that the other formula () is not valid in this case. This is because the roll of a die occurs first. This means is independent. The number of tosses depends on the value of , making dependant.

The roll of a die is simple enough.

is a binomial distribution, since we want to find the number of heads (success) in attempts.

b)

We can find all of these values by using a matrix.

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6.

Let and .

The first thing to understand is that the first run cannot be of length . If we say that the first run consists of s and is of length , the that is not the first run at all. In that case, the first run consists of s not s.

If we know that the first run consists of a certain digit, we want to find the expected number of attempts needed to get a ‘success’, i.e. get the opposite digit. Now, what if we get the opposite digit in the first attempt? That would mean, by our definition, the first run is of length , which we just saw cannot be true. This means, we cannot start counting at the first digit. We have to start counting the number of attempts needed to get the opposite digit starting from the second digit, since we already have the first digit.

The number of attempts to get a success is just a geometric distribution. Thus,

Here, we subtracted a , since the original formula for a geometric distribution gives us the number of attempts until a success occurs, including the attempts for the success. We want the length, which is less than the success. We added a since we started counting from the second attempt, but the length should include the first attempts.